## COMP3151/COMP9151

## Foundations of Concurrency

Time allowed: 2 hours (8:45-11:00)
Total number of questions: $\mathbf{5}$
Total number of marks: 45
Textbooks, lecture notes, etc. are not permitted, except for 2 double-sided A4 sheets of hand-written notes.

Calculators may not be used.
Not all questions are worth equal marks.
Answer all questions.
Answers must be written in ink.
You can answer the questions in any order.
You may take this question paper out of the exam.
Write your answers into the answer booklet provided. Be concise - excessively verbose answers will be penalised. Use a pencil or the back of the booklet for rough work. Your rough work will not be marked.

## Shared-Variable Concurrency ( 15 Marks $\approx 40$ minutes)

Recall that starvation-freedom is the liveness property relevant to mutual exclusion algorithms. It is satisfied if every process trying to enter its critical section will eventually do so.

## Question 1 (10 marks)

Let $A$ and $B$ be two algorithms which were designed to solve the mutual exclusion problem, and let $C$ be the algorithm obtained by replacing the critical section of $A$ with the algorithm $B$ :

|  | Algorithm: $C$ ( $n$ processes) |
| :--- | :--- |
|  | shared vars of $A$ <br> shared vars of $B$ |
| loop forever |  |
| p1: | non-critical section |
| p2: | entry protocol of $A$ |
| p3: | entry protocol of $B$ |
| p4: | critical section |
| p5: | exit protocol of $B$ |
| p6: | exit protocol of $A$ |

Assume that the shared variables of $A$ are disjoint from those of $B$. Are the following statements correct? Justify each answer briefly (i.e., with a sentence or two).
(a) If both $A$ and $B$ are deadlock-free then $C$ is deadlock-free.
(b) If both $A$ and $B$ are starvation-free then $C$ is starvation-free.
(c) If either $A$ or $B$ satisfies mutual exclusion then $C$ satisfies mutual exclusion.
(d) If $A$ is deadlock-free and $B$ is starvation-free then $C$ is starvation-free.
(e) If $A$ is starvation-free and $B$ is deadlock-free then $C$ is starvation-free.

## Question 2 (5 marks)

Does the following mutual exclusion algorithm satisfy starvation-freedom? Sketch a proof or present a counter-example.

| Algorithm: algorithm \#3 |  |  |  |
| :---: | :---: | :---: | :---: |
| bit array $\mathrm{b}[0 . .1] \leftarrow[0,0]$ |  |  |  |
|  | p |  | q |
|  | op forever |  | op forever |
| p1: | non-critical section | q1: | non-critical section |
| p2: | $\mathrm{b}[0] \leftarrow 0$ | q2: | $\mathrm{b}[1] \leftarrow 1$ |
| p3: | while $\mathrm{b}[0]=0$ do | q3: | while $\mathrm{b}[0]=1$ do |
| p4: | $\mathrm{b}[0] \leftarrow 1$ | q4: | $\mathrm{b}[1] \leftarrow 1$ |
|  | while $\mathrm{b}[1]=1$ do $\mathrm{b}[0] \leftarrow 0$ od od | q5: | while $\mathrm{b}[1]=0$ do $\mathrm{b}[1] \leftarrow 0$ od od |
| p6: | critical section | q6: | critical section |
| p7: | $\mathrm{b}[0] \leftarrow 0$ | q7: | $\mathrm{b}[1] \leftarrow 0$ |

## Message-Passing Concurrency ( $\mathbf{3 0}$ Marks $\approx \mathbf{8 0}$ minutes)

## Question 3 (8 marks)

Hamming's problem. Use transition diagrams to present a message passing concurrent program $P=P_{2}\left\|P_{3}\right\| P_{5} \| M$ whose output along channel Out is the sequence of all multiples of 2, 3, and 5 in strictly ascending order. The first elements of the sequence are $0,2,3,4,5,6,8,9$, $10,12,14$. There will be four concurrent processes: one $P_{i}$ each to calculate the multiples of the numbers $i=2,3$, and 5 , respectively, and a fourth process $M$ to merge the results.

## Question 4 (12 marks)

Modify your solution $P$ to Hamming's problem such that it terminates after $k$ numbers have been sent to channel Out, where $k \in \mathbb{N}$ is a constant known to the merger process. (2 marks)

Define a post-condition $\psi$ for $P$ to capture the essential properties of $P$ as specified above. (2 marks)

Outline a proof of $\{$ true $\} P\{\psi\}$ (8 marks).

## Question 5 (10 marks)

Recall the Ricart-Agrawala distributed mutual exclusion algorithm:

| Algorithm: Ricart-Agrawala algorithm |  |
| :---: | :---: |
|  | integer myNum $\leftarrow 0$ <br> set of node IDs deferred $\leftarrow$ empty set <br> integer highestNum $\leftarrow 0$ <br> boolean requestCS $\leftarrow$ false |
| Main |  |
| loop forever |  |
| p1: | non-critical section |
| p2: | requestCS $\leftarrow$ true |
| p3: | myNum $\leftarrow$ highestNum +1 |
| p4: | for all other nodes N |
| p5: | send(request, $\mathrm{N}, \mathrm{myID}$, myNum) |
| p6: | await reply's from all other nodes |
| p7: | critical section |
| p8: | requestCS $\leftarrow$ false |
| p9: | for all nodes N in deferred |
| p10: | remove N from deferred |
| p11: | send(reply, $\mathrm{N}, \mathrm{mylD}$ ) |
| Receive |  |
| integer source, requestedNum loop forever |  |
| p12: | loop forever receive(request, source, requestedNum) |
| p13: | highestNum $\leftarrow \max ($ highestNum, requestedNum) |
| p14: | if not requestCS or requestedNum < myNum |
| p15: | send(reply, source, myID) |
| p16: | else add source to deferred |

(a) 4 marks: Construct a scenario in which the ticket numbers are unbounded.
(b) $\mathbf{2}$ marks: Can the deferred lists of all the nodes be non-empty?
(c) $\mathbf{2}$ marks: What is the maximum number of entries in a single deferred list?
(d) 2 marks: What is the maximum number of entries in all the deferred lists together?

Justify your answers to (b)-(d) briefly (i.e., with a sentence or two).

